# Dynamics of granular band formation: Long-term behavior in slurries, parameter space, and tilted cylinders

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Band formation (axial segregation) and subsequent coarsening of bidisperse mixtures in long circular tumblers is well documented for the case where the cylinder is at a single fill level and the interstitial fluid is air. However, little information is available for a range of fill levels, nor is the effect of rotational speed on segregation clear. Moreover just a handful of studies have focused on slurry systems, where the interstitial fluid is a liquid. This is precisely the parameter space covered in this study. Experiments are conducted using a 2:1 mixture of 882 and 272 $\mu$m glass beads with water as the interstitial fluid. Several different phenomena are uncovered. Results indicate that bands are less likely to form at low rotational speeds and low fill levels. As the fill level and rotational speed increase, more bands form and they form more quickly. However, at fill levels near 50% and high rotational speeds the bands contain a mixture of particles rather than being relatively pure. Furthermore, the evolution of the core of small beads that forms deep in the bed depends on the fill level and the rotational speed. For certain fill levels and rotational speeds, the core remains prominent as bands form, while in other cases the core disappears entirely between bands. Finally, when the tumbler is tilted so that the fill level varies from 14% at one end to slightly more than half full at the other end, the bands and core that form locally qualitatively correspond with those that would form for the corresponding fill level in a horizontal cylinder.

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## I. INTRODUCTION

Flowing granular mixtures tend to segregate when there are differences in particle properties such as density or size. In partially filled rotating tumblers the flow drives segregation perpendicular to the axis of rotation (radial segregation) due to percolation for size differences or buoyancy for density differences. In longer tumblers particles of different properties may also evolve into a configuration comprising what appear at the surface of the tumbler to be alternating bands rich and lean in one of the components (axial segregation). This type of segregation is of considerable practical interest, since it affects the performance of equipment such as drum mixers and rotary kilns, which usually operate with a tilt to induce axial flow. Since the discovery of the phenomenon by Oyama [1], primarily in the last ten years, several aspects of the onset and dynamics of axial segregation have been studied; detailed reviews are given by Stavans [2] and Levine [3].

With only a few exceptions, nearly all of the voluminous research on granular flows during the last decade has focused on dry granular systems, where particles are immersed in air or are in a vacuum. Only a handful of studies have addressed the motion of granular materials in wet granular systems (slurries), where particles are completely immersed in a less dense liquid [4,5]. It is important to stress that slurries are important in their own right with a wide range of practical industrial applications (e.g., preparation of pharmaceuticals and foodstuffs). Furthermore, there are many environmental and geophysical situations involving slurry or slurrylike flow (e.g., mudslides and sediment transport). Although we consider a slurry situation here, the focus is on the nature of the granular band formation, which occurs whether the interstitial fluid is a gas or a liquid. The immediate value of the results is that they pose a challenge for existing theories of axial segregation.

### A. Mechanisms and qualitative theory

Axial segregation and the resulting pattern formation are counterintuitive because there is no obvious driving force for the axial motion in the system. Radial segregation occurs very quickly, often within a single rotation of the tumbler. With more rotations (10–100 turns), the particles separate further into axial bands, and complex dynamics of band merging may ensue. Magnetic resonance imaging and other visualization studies of the internal structure of axially segregated systems have shown that the radial core does not disappear, but becomes varicose, contracting in the region of the large particle bands and expanding to fill the cross section in the region of the small particle bands [4,6,7]. These studies also reveal that the bands of small particles are actually a mixture of the small and large particles, whereas the bands of large particles are relatively pure with a segregated core of the small particles.

One prevalent theory is that axial segregation due to differences in particle size is a result of a difference in the angles of repose of the two granular materials of a bidisperse mixture [8–10]. Because of more efficient packing, the mixed phase of particles has a higher density than monodispersed large particles, so it can sustain more shear leading to a higher dynamic angle of repose than the monodispersed large particles. After the initial radial segregation, the two types of particles continually remix in the flowing layer with the smaller particles percolating downward through the flowing layer to reform the core. Randomness in the system or a slight variation in the relative concentration of the particles can lead to a local increase in the dynamic angle of repose. In either case, when this occurs, larger particles, which tend to be near the surface of the flowing layer due to percolation and radial segregation, will tumble along the line of steepest descent, which has a slight axial component due to the locally higher angle of repose of the mixed phase. This reduces the concentration of large particles in the band in the flowing layer where the angle of repose is higher and increases the concentration of large particles in the adjacent band. Since the larger particles have a lower angle of repose than the mixed particles, this process continues, bringing more large particles into the band with a low angle of repose while depleting the mixed band of large particles. Of course, the band of large particles that is forming is adjacent to mixed particles on the side opposite that of the initial band of mixed particles. Since the angles of repose are different, large particles from this opposite side also enrich the large particle band and deplete the mixed band of large particles leading to another local increase in the angle of repose. Thus, the band formation propagates to either side of the initial disturbance. Eventually the entire system fills with alternating bands of large and small particles visible at the surface of the granular bed. Beneath the surface a core of small particles remains due to radial segregation.

### B. Onset and long-time behavior

Several experimental studies have investigated the effect of operating conditions on band formation (onset behavior). Fewer studies have focused on long-time behavior, including coarsening. Das Gupta *et al.* [9] found that the onset of axial segregation depends on rotational speed for dry mixtures of sand of different sizes. They found that axial segregation occurred only for rotation rates greater than a critical value when the difference between the angles of repose of the pure components was sufficiently large. Hill and Kakalios [11] found a critical rotational speed above which axial segregation occurred for some mixtures of glass beads, and the axial segregation was reversible. The condition for segregation was found to be a sufficient difference between the angles of repose for the mixture and the larger particles. Choo *et al.* [12] found that waves could emerge during the onset of band formation for certain wavelengths. Hill *et al.* [13] studied the effect of fill levels on onset behavior in tumblers with circular and square cross sections for a narrow range of fill levels near half full. They found that remnants of the segregation patterns seen in quasi-two-dimensional tumblers at various fill levels are still present in the longer tumblers with corresponding fill levels and that the effect of the fill level is very important in the case of square cross sections and much less so in the case of circular tumblers.

Several experimental studies have also considered the long-term dynamics of bands. Nakagawa [14] and Hill *et al.* [15] found merging of bands with increasing rotations. Jain *et al.* [4] found that axial bands form more quickly in slurries than in dry granular flows. Fiedor and Ottino [16] found that when bands did merge the number of bands decayed logarithmically. They also found traveling waves exist at faster rotation rates in dry systems after bands are fully developed.

To date, with the exceptions noted above, the bulk of the studies of axial segregation and band formation has focused on dry systems (air being the interstitial fluid) at a single fill level, usually near 30% [12,15] or half full [6,13]. However, many practical situations involve fill levels that are much less than half full as well as cylinders that have a varying fill level due to the cylinder being at an angle with respect to horizontal. In this study, we consider axial segregation of slurry systems in cylinders with circular cross sections that are less than half full and in tilted cylinders to examine the impact of the fill level on the formation and long-term dynamics of band formation, along with the internal structure of the granular bed.

## II. EXPERIMENTAL METHODS

The experimental setup consisted of a horizontal tube that was partially filled with two different sized glass beads rotating at different speeds. The acrylic tube had a diameter of 6.35 cm and was 0.75 m long. It was rotated about its axis by a dc stepper motor with a planetary gear drive. The small glass beads were $272 \pm 24$ $\mu$m black beads, while the large glass beads were $882 \pm 30$ $\mu$m clear beads. The density of both beads was 2.5 g/cm$^3$. The volume ratio of small to large beads was held constant at 1:2 for all experiments. In dry systems axial segregation and the subsequent coarsening take place over several hundred rotations [16]. In liquid systems the process is accelerated. For this reason water was used as the interstitial fluid. A few drops of detergent were added to the water to reduce the surface tension, so the smaller beads would not get caught in the air-water interface of the trapped air bubbles in the tube.

The tube was filled with a mixture of small and large beads to levels between 20% and 50% in 5% intervals (seven total). Here the fill level is defined as the fraction of the tube volume that appears filled with particles when the tube is in a vertical orientation. This definition is somewhat different from that often used for quasi-two-dimensional granular flow studies where the fill level is defined as the depth of the granular material as a fraction of the diameter of the tumbler [17]. Care was necessary in filling the tube with the desired fraction of beads because the mixed beads packed more tightly than unmixed beads. First, the tube was held vertically and filled with tap water. Then large beads were added to the desired fill level, followed by adding the small beads in the proper proportion. Then the tube was sealed and rotated to a horizontal position. The tube was shaken horizontally in the direction perpendicular to the axis of the tube to mix the beads. The tube was then rotated back to a vertical position very slowly to prevent segregation as the beads flowed to the lower end of the tube. More beads were added in the proper proportion. Usually two or three iterations of this process were necessary to reach the desired percentage fill level to within ±0.8%.

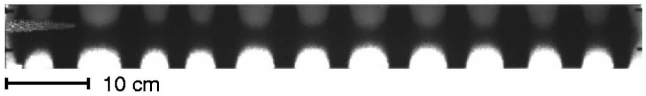


FIG. 1. Image of an experimental run cropped so that only the granular bed is shown. Smaller particles (dark) form a core as well as bands between bands of larger particles (white and gray). Illumination is from the bottom of the tumbler.

Achieving a homogeneous mixture of small and large beads as an initial condition is quite difficult because of the propensity of the beads to segregate. For fill levels $\geqslant 35\%$ the tube was rotated at high speed (130 to 170 rpm) to achieve a homogeneous mixture. For lower fill levels, this method did not provide a uniform mixture, so the tube was shaken horizontally to mix the beads.

Images of the tube were obtained using a Kodak Megapixel 1.4i digital camera positioned approximately 4 m from the apparatus to ensure that the image included the entire length of the tube. Images were obtained from above the horizontal tube. By synchronizing the camera to the stepper motor, an image was obtained at the same tumbler position one time per revolution. A fluorescent light was positioned parallel to the tube to illuminate the flowing surface. Because the flowing surface was angled with respect to horizontal, the images obtained from above the tube are of the angled surface, an example of which is shown in Fig. 1. The small black particles appear black in the image. The large clear particles appear white or gray in the image due to the variation in the lighting. Although the water does not match the index of refraction of the glass beads, the use of clear beads nevertheless permits the visualization of a core of smaller black beads when such a core occurred, similar to that observed in previous index-matched experiments [4]. The slight distortion at the left end of the image is a result of a small amount of air in the tube. Although every attempt was made to mount the tube horizontally, the appearance of the bubbles at one end of the tube indicates that the tube is not perfectly horizontal. In this particular case, fairly regular bands of large beads (light colored) and small beads (dark colored) appear at the visible surface of the granular bed (evident as alternating light and dark bands along the upper and lower edges of the image). A continuous core of small beads is evident below the surface of the granular bed (along the horizontal centerline of the image).

The evolution of the axial segregation process was tracked by using space-time plots. In these plots, images of the entire tube were cropped to form a modified image having the length of the tumbler and the width of a single pixel line. The slice was taken at the lower edge of the flowing layer (bottom edge of the image in Fig. 1) to assure that only the band structure was recorded without interference of the central core of darker beads below the axis of the tube. Since the time scale of the motion of axial bands is much longer than the period of one tumbler revolution, a single row of pixels is sufficient to represent the band structure in the tumbler. The cropped images were stacked on top of one another to form a space-time series over 2000 revolutions of the tumbler like that shown in Fig. 2 with time progressing from top to bottom. The narrow, horizontal gray zone at the top of Fig. 2 represents mixed particles, although a core of small particles forms after one or two revolutions. After about 50 revolutions (depending on the fill level), bands begin to form with band merging occurring for several hundred revolutions thereafter. In some cases, a space-time plot did not accurately represent the structure of the segregation in the tumbler. As a result, in some cases we also present images of the entire tumbler as viewed from the top.

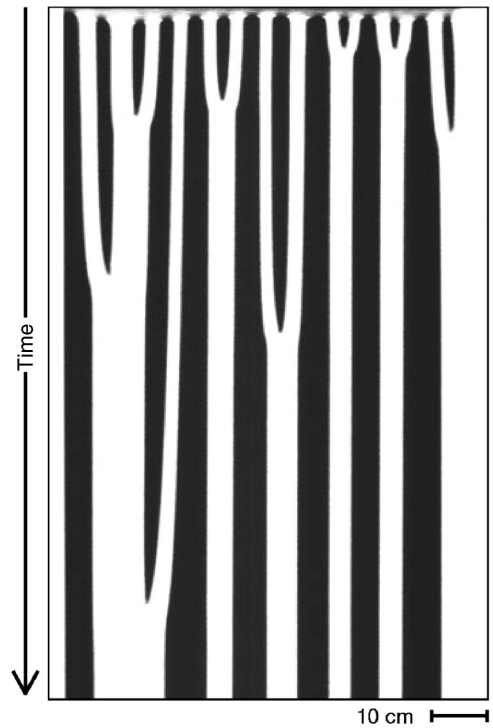


FIG. 2. Example of a space-time series (2000 revolutions from top to bottom). Cropped images are appended atop each other to show the band dynamics of an entire experimental run, as seen at the surface of the granular bed.

Experiments were performed over rotational speeds of 2–26 rpm, corresponding to a Froude number, $\mathrm{Fr}=\omega^2 R/g$, of $1.4\times10^{-4}\leqslant\mathrm{Fr}\leqslant 2.4\times10^{-2}$, where $\omega$ is the rotational speed, $R$ is the radius of the tumbler, and $g$ is the acceleration of gravity. The fill level was varied from 20% to 50% in 5% increments. Experiments for fill levels below 20% yielded inconsistent results, most likely due to the sensitivity to the initial degree of mixing and the difficulty in obtaining a homogeneously mixed initial state for low fill ratios. Since fill ratios greater than 50% result in an unmixed core of material [17], the upper bound for the fill ratio was set at 50%. Of course, when the tube is filled to less than 50%, the length of the flowing layer is less than $2R$. It is clear that the Froude number as defined above does not account for the fill level in the tumbler. Therefore, we define a modified Froude number, $\mathrm{Fr}_{mod}=\omega^2 L/g$, where $2L$ is the streamwise length of the flowing layer. Thus, the modified Froude number is equal to the Froude number for a 50% fill level and less than the Froude number for lower fill levels. For these experiments, the modified Froude number was in the range $1.2\times10^{-4}\leqslant\mathrm{Fr}_{mod}\leqslant 2.4\times10^{-2}$.

For a limited number of experiments, the tube was set up at a 1.4° angle with respect to horizontal. In these cases the

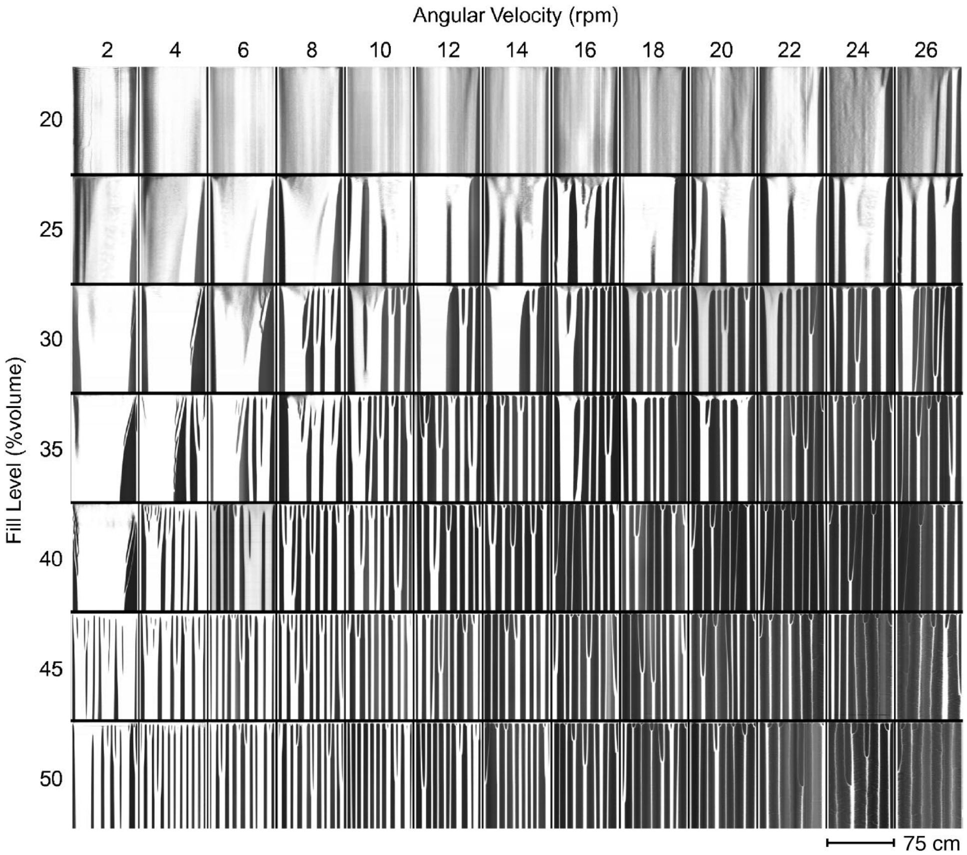


FIG. 3. Phase portrait of space-time plots as a function of the angular velocity and the fill level.

tube was filled as described previously. However, during the shaking to obtain a uniform mixture, the tube was tilted at approximately this angle as it was shaken horizontally to mix the particles. In this way, a nearly homogeneous mixture of beads was obtained over the length of the tube even though the fill level varied from 14% at the shallow end to 57% at the deep end. The inclined experiments were performed at 8, 16, and 24 rpm.

## III. RESULTS

The results of the experiments are presented in the following manner. First, we present a phase plot of the space-time diagrams as a function of both fill level and angular velocity for axial segregation in granular slurries based on bands as seen at the surface of the granular bed. Next, results based on subsurface observations, mainly looking for the existence of a radial core consisting of smaller particles within bands of larger particles, are described. Then these results are compared to results of experiments run in a similar system that is slightly inclined resulting in a varying fill level along the length of the tumbler.

### A. Phase plot fill level and angular velocity

The pattern of axial bands and how they emerge varies substantially with the rotational speed [9–11] and with fill level. The most compact way to display this dependence is in terms of a phase portrait, or matrix, of the space-time diagrams, shown in Fig. 3. These results are based on axial bands that appear near the surface of the granular bed. Each space-time diagram in the figure represents 2000 revolutions, just like that shown in Fig. 2. The angular velocity is varied along the horizontal axis (from 2 to 26 rpm) and the fill level is varied along the vertical axis (from 20% to 50% full). This results in $\mathrm{Fr}_{mod}$ ranging from $1.2\times10^{-4}$ (at 20% full and 2 rpm) to $2.4\times10^{-2}$ (at 50% full and 26 rpm).

Some trends are clearly evident. As either the fill level or the angular velocity increases, the tendency for band formation increases. At low fill levels, bands are less likely to form, bands start to form after a greater number of revolutions, and fewer bands are formed if bands are present. On the other hand, at high fill levels, many bands form very quickly. Two types of behaviors are clearly evident in the matrix of space-time plots shown in Fig. 3. At lower fill levels and slower rotation rates bands tend to form only near the ends of the tumbler. In dry systems this is where bands usually appear first [16], apparently due to the high shear near the ends of the tumblers. At higher fill levels and faster rotation rates bands tend to form at the same time throughout the tumbler. There is still a region of higher shear near the ends, but in this case the interaction of particles in the flowing layer must be responsible for the band formation, not the shear near the end walls. The transition for how the bands form occurs at fill levels between 20% and 40%. At lower rotation rates the bands form due to end effects. At faster rotation rates the bands form across the entire flowing layer. At lower fill levels the rotation rate needs to be faster for bands to form independent of the end walls.

When bands form throughout the entire tumbler, "typical" band dynamics characteristic of previous experiments at a

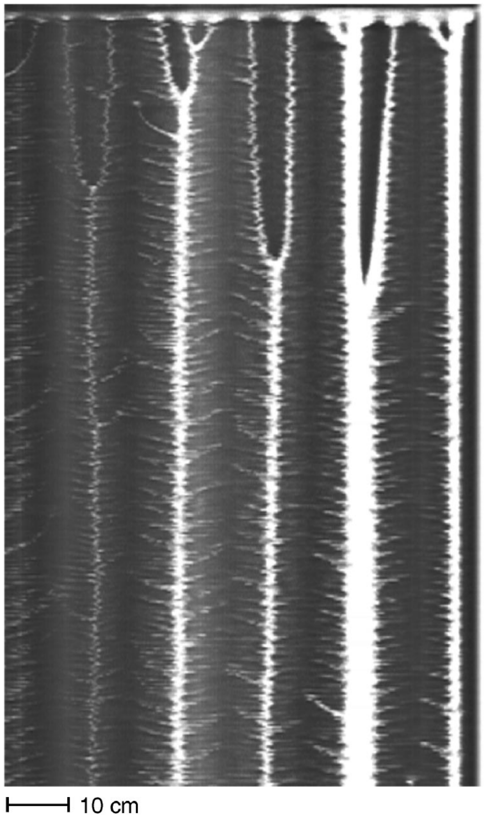


FIG. 4. Experimental space-time series for 26 rpm and 45% fill level for the first 400 revolutions. The "fuzzy" interfaces between dark and white bands are "wavy" or "unstable."

50% fill level always occur. That is, new bands do not form after a short initialization period and thinner bands that initially form tend to merge into wider bands. There is no evidence of standing or traveling waves as previously seen during the onset of band formation [12] or on longer time scales with bands fully developed [16]. However, such traveling waves have appeared only in dry systems, and not in any studies of slurries. At faster rotation rates in near half-full cylinders, the interfaces between dark and light bands become "wavy" or "unstable," as shown in Fig. 4, as a result of varying precession of the sides of the dark bands. The interfaces between bands meander around an average axial position resulting in the "fuzzy" appearance of the space-time diagrams. The meandering length can be as much as two orders of magnitude greater than the particle diameter in the axial direction. At slower rotation rates this does not occur. Instead the interfaces are almost stationary, appearing quite "sharp," as shown in Fig. 2, and only shift axially when bands merge.

### B. Band dynamics

Quantitative data are extracted from the space-time series to study the band dynamics. In particular, we look at the area of dark bands and the number of dark bands, which are rich in smaller particles. The data include only bands that reach the surface of the granular bed. Subsurface bands are not included. Thus a system with higher area coverage than an-

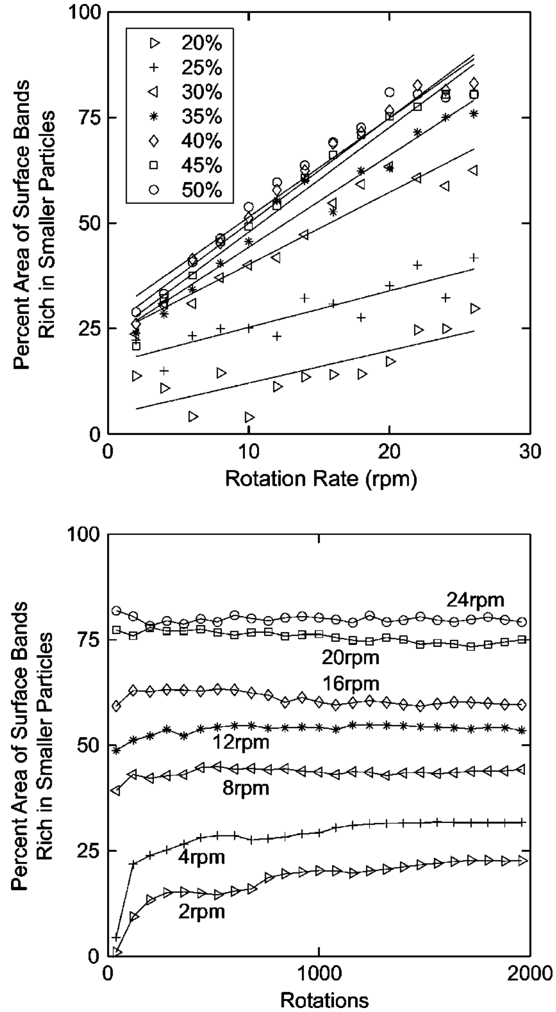


FIG. 5. Top: The effect of rotation rate on the area of bands rich in smaller particles (dark bands in the experiments) after 2000 revolutions. The lines are least squares fits of the data for various fill levels. Bottom: The evolution of the area of bands rich in smaller particles over time for a fill level of 40%.

other could indicate one of two situations. Either the dark bands rich in smaller particles are more contaminated with larger particles or the core is deficient in smaller particles. Determining which of these situations occurs would require further study.

In general, the area of dark bands, which are rich in smaller particles, increases with rotation rate for all fill levels (see top plot of Fig. 5). Higher fill levels also exhibit more area coverage than lower fill levels for the same rotation rate. For high fill levels, the area of the dark bands ranges from about 30% at 2 rpm to about 80% for 26 rpm. For lower fill levels (20% and 25%) the area of the bands remains much lower, regardless of the rotation rate, consistent with the appearance of the bands only near the ends of the tumbler. An attempt was made to collapse the data in the top portion of Fig. 5 by plotting the percent area of the surface bands rich in smaller particles as a function of the modified Froude number, which accounts for both the rotational speed and fill level. However, the data did not collapse when plotted in this format, so clearly the modified Froude number does not ac-

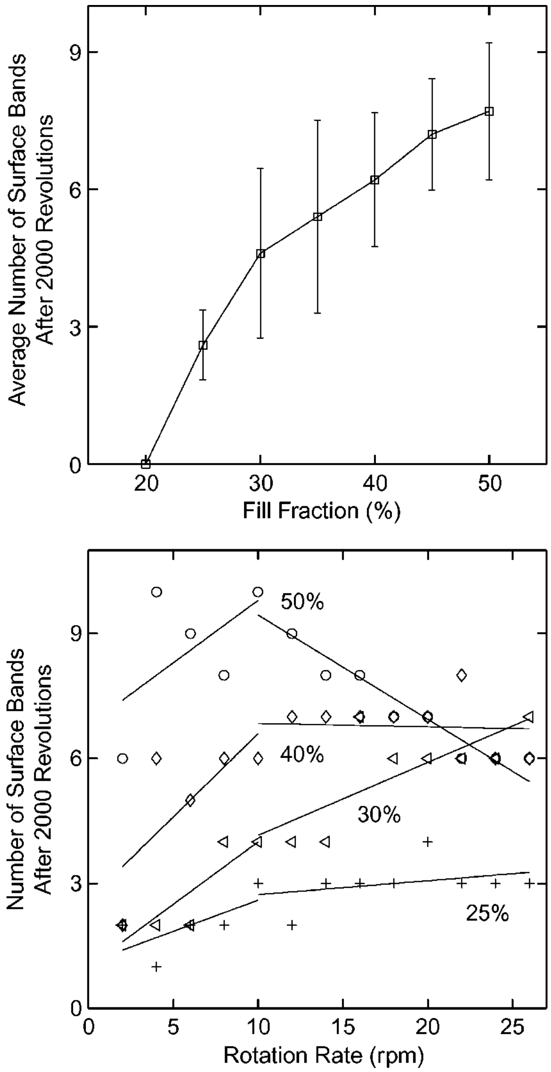


FIG. 6. The effect of fill level and rotation rate on the number of dark bands (rich in smaller particles). The upper plot averages the number of bands over all rotation rates after 2000 revolutions (error bars correspond to plus or minus one standard deviation). The lower plot shows that lower fill levels result in few bands and higher fill levels result in many. The lines are two linear fits at different ranges of rotation rates for each fill level.

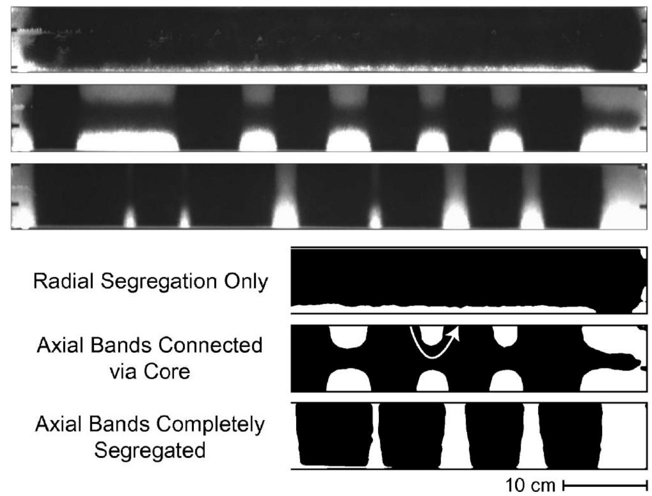


FIG. 7. Top three images are of experiments showing the three types of core behavior. Top image: Radial segregation with no banding. Second image: Banding with continuous central core connecting the bands. Third image: Banding with no core between bands (completely separated bands). The bottom three images are axially cropped and thresholded images of the top three images to emphasize where a core exists and does not exist. The arrow in the second to last image indicates the flow of small particles when a band disappears by merging into an adjacent band.

count for these variables. The area of dark bands remains constant after bands initially form, particularly for faster rotation rates (see bottom plot of Fig. 5). At slower rotation rates (2–4 rpm) the area of dark bands continues to slowly increase over time after an initial sharp increase. The plot only shows results from experiments run at a fill level of 40%, but similar results occur for all fill levels.

The number of bands rich in smaller particles also increases with fill level, as shown in the top plot of Fig. 6. In this figure, the number of bands after 2000 revolutions is averaged over all rotation rates. The increase in the number of bands is consistent with the phase portrait of space-time plots; fewer bands form at lower fill levels. The bottom plot of Fig. 6 shows the effect of rotation rate on the number of bands after 2000 revolutions. The presentation of these results presents difficulties. The number of experiments contained in the figure is large (52 experiments requiring a total of 66 h) and though repetition of runs would make trends more clearly discernible, this is impractical in the context of this already experiment-packed study. The interpretation is however clear, no matter what type of fitting methodology is selected—two linear fits, one quadratic, etc. The slowest rotation rate (2 rpm) has the fewest bands and the number of bands generally increases with rotational speed. Lower fill fractions (20%–40%) exhibit an increase in the number of surface bands with increasing rotational speed. As the fill fraction approaches 50% the number of surface bands initially increases with rotational speed and then decreases.

Using the data from Figs. 5 and 6, the average width of the bands rich in smaller particles after 2000 revolutions was calculated. Taking into account all experiments, the average band width is about 250 particle diameters (1.2 tumbler diameters). At 40%, 45%, and 50% fill levels the average band width ranges from 100 to 400 particle diameters (0.5 to 1.9 tumbler diameters) and increases with rotation rate. At the other fill levels (20%–35%) the average band width did not change with rotation rate, but rather fluctuated around the overall average band width (250 particle diameters).

### C. Subsurface dynamics

The space-time series used in this study only give information about what is occurring at the surface of the system. In order to study the evolution of the segregation in more detail, we also examine the core of the system by considering the entire image of the illuminated system. Figure 7 shows examples of the three different core structures that can exist in the tumbler: radial segregation without banding, axial

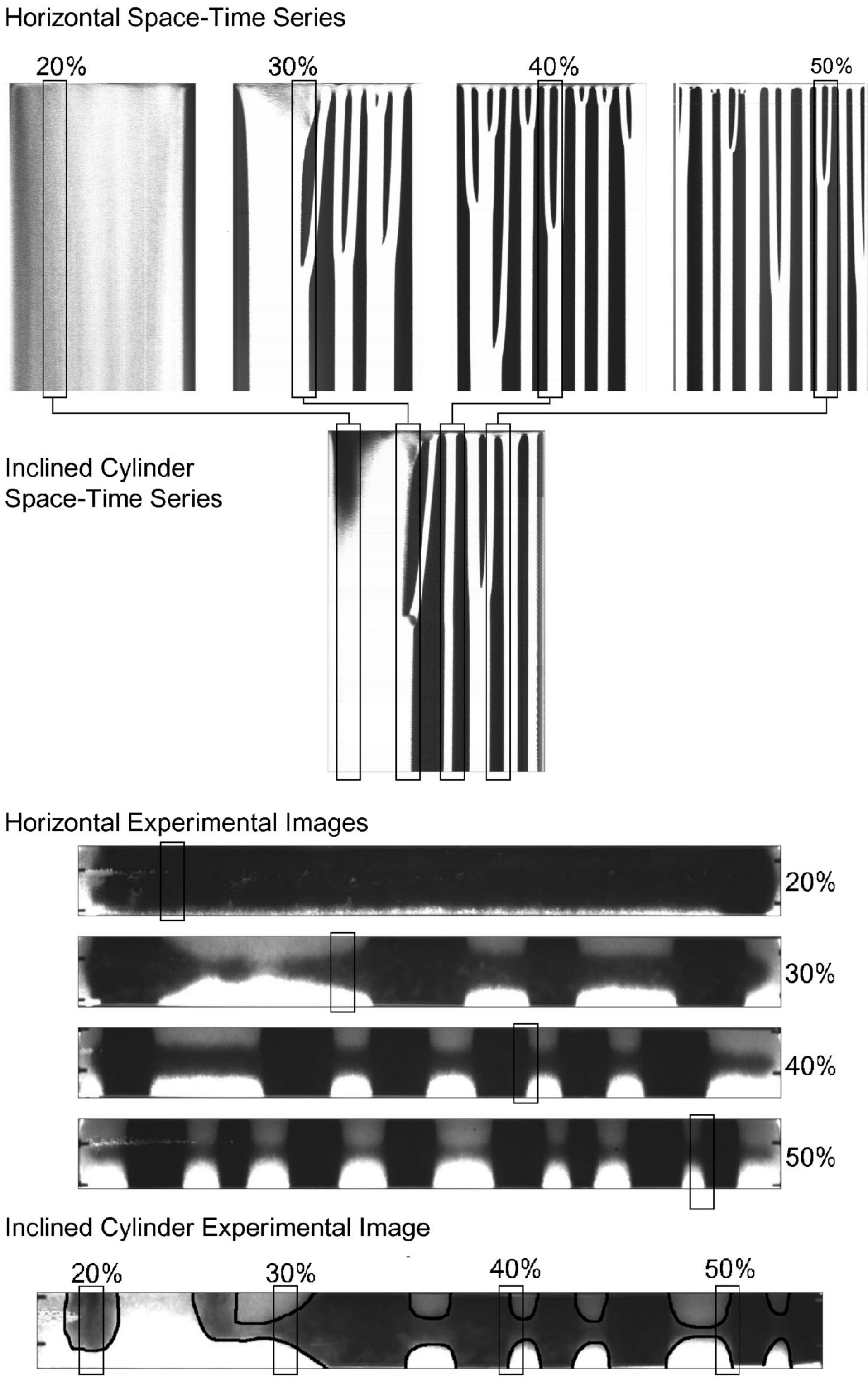


FIG. 8. Comparison of tilted cylinder experiments with a rotation rate of 8 rpm to horizontal experiments at the same rotation rate. The left end of the cylinder has the lowest fill level. Top: Space-time series for the horizontal experiments at various fill levels compared to the inclined experiment. Bottom: Images of horizontal experiments for the same fill levels compared to the inclined experiment.

banding with an interconnecting radial core, and completely segregated axial bands with breaks in the core. The top image of Fig. 7 shows a situation with radial segregation only. Toward the bottom of the image is a region of clear, larger particles that have segregated around the core of small beads. A single band that reaches the lower surface of the image is evident on the right end. The second image shows bands that have formed with a continuous core of smaller dark beads connecting all of them. The third image shows bands that have formed with no connecting core. The bands are completely segregated from each other.

Previous studies looking at subsurface dynamics in axial segregation have found that bands rich in smaller particles are connected via a continuous radial core [6,7,16]. Most interaction between the bands occurs via this core. For instance, when two bands merge, particles from one band move down into the core while particles in the core near the other band move out from the core to the surface. For fill levels greater than 30% and rotation rates less than 14 rpm this is indeed the case. Bands form after some initialization period with a continuous radial core connecting them. Over time some of these bands merge together, but a radial core between bands always remains.

For fill levels greater than 30% and rotation rates of 14 rpm and greater, bands still form but some are completely separated having no radial core between them. Systems may simultaneously contain bands that are connected via a core and bands that are completely separated. The faster the rotation rate and the higher the fill level, the more likely bands will form without the radial core. But faster rotation rates also increase the width of the bands rich in smaller particles. In some instances it is difficult to determine if a core exists beneath the surface connecting the bands because two wide bands are nearly in contact. Also for the fastest rotation rates in nearly half full tumblers bands become "wavy," as shown in Fig. 4, making it difficult to identify a core.

For fill levels less than 30%, all three types of behavior can occur. Radial segregation initially occurs throughout the tumbler for all experiments, even with fill levels more than 30%, before any bands begin to form. But in experiments with fill levels less than 30%, the radial segregation without bands persists for much longer times (at least 100 revolutions compared to around 30 revolutions for higher fill levels). Bands eventually form with a continuous radial core connecting them. Over time the core disappears between some of the bands. At the fastest rotation rates (over 16 rpm) the core begins to disappear within 1000 revolutions; for some experiments (16 and 24 rpm) the core begins to disappear even sooner, within 500 revolutions. For some slower rotation rates (6 and 8 rpm) the core disappears within 2000 revolutions, while for others the continuous radial core persists for the duration of the experiment.

### D. Tilted cylinders

A single tilted cylinder results in a continuum of fill levels. Tilted cylinder experiments were run at 8, 16, and 24 rpm. The central question is to what extent the "local" results corresponding to a given fill level in the tilted cylinder correspond to the experiment with the same fill in a horizontal cylinder. Figure 8 shows the results for the inclined experiment run at a rotation rate of 8 rpm and also results from the horizontal experiments at the same rotation rate for various fill levels (20%, 30%, 40%, and 50%). Both space-time series and experimental images are used to compare the band dynamics along with the core structure. In the inclined cylinder experiment there is a region of radial segregation only for lower fill levels and a region of axial bands connected with a continuous core at higher fill levels. Near the end with the lower fill level bands do not form, while at the other end bands form and then merge. Comparing the inclined experiment at different axial positions to the corresponding horizontal experiments shows that the local fill level determines the local behavior in the tilted cylinder. This is true for both the band dynamics and the core structure.

The same results hold true for the inclined experiments at 16 and 24 rpm—the local fill level determines the local behavior. At 16 and 24 rpm all three types of core behaviors are observed depending on the local fill level. Near the end with the lower fill level no bands form for both experiments (see Fig. 9). Near the end with the higher fill level bands form and merge for the experiment run at 16 rpm. For the experiment run at 24 rpm, bands form near the end with the higher fill level, but none merge during the experiment (although two bands would probably merge if the experiment was run long enough for the merging to complete). There is even evidence of "wavy" or "unstable" bands at the end with the higher fill level, similar to those seen in Fig. 4, in the experiments run at 24 rpm.

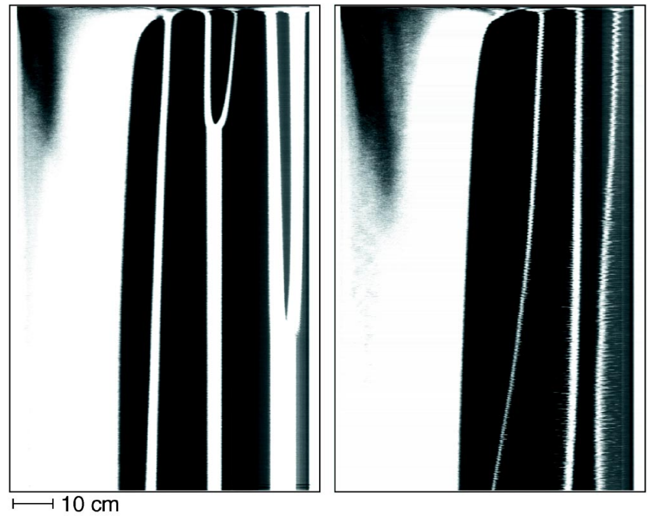


FIG. 9. Space-time series for tilted cylinder experiments with rotation rates of 16 rpm (left) and 24 rpm (right). The left end of the cylinder has the lowest fill level.

## IV. CONCLUSIONS

A variety of axial banding mechanisms occur in slurry systems rotated in long circular tumblers. This is clearly an area where experiments are in need of a clarifying theory. We have demonstrated that both fill level and rotation rate have an effect on the band dynamics and the core structure. Bands are less likely to form at lower fill levels and slower rotation rates. When bands do form, they tend to be near the end walls where there is a region of higher shear. As the fill level and rotation rate increase, more bands form and they form more quickly. Also the bands form throughout the tumbler, not just near the end walls. Furthermore, the evolution of the core of small beads that forms deep in the bed depends on the fill level and the rotational speed. For certain fill levels and rotational speeds, the core remains prominent as bands form, while in other cases the core disappears entirely between bands. Finally, when the tumbler is tilted so that the fill level varies from 14% at one end to approximately half full at the other end, the bands and core that form locally are consistent with those that would form for the corresponding fill level in a horizontal cylinder.

Several issues await clarification. One general issue is the phenomenon of "wavy" bands. What causes this "instability" and when does it occur? Another is an extension of these studies to cylinders with noncircular cross sections. Experiments already indicate that fill level has a large impact on the radial segregation pattern in tumblers with square cross sections [13]. It would be interesting to see how a square cylinder system with a varying fill level would behave.

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